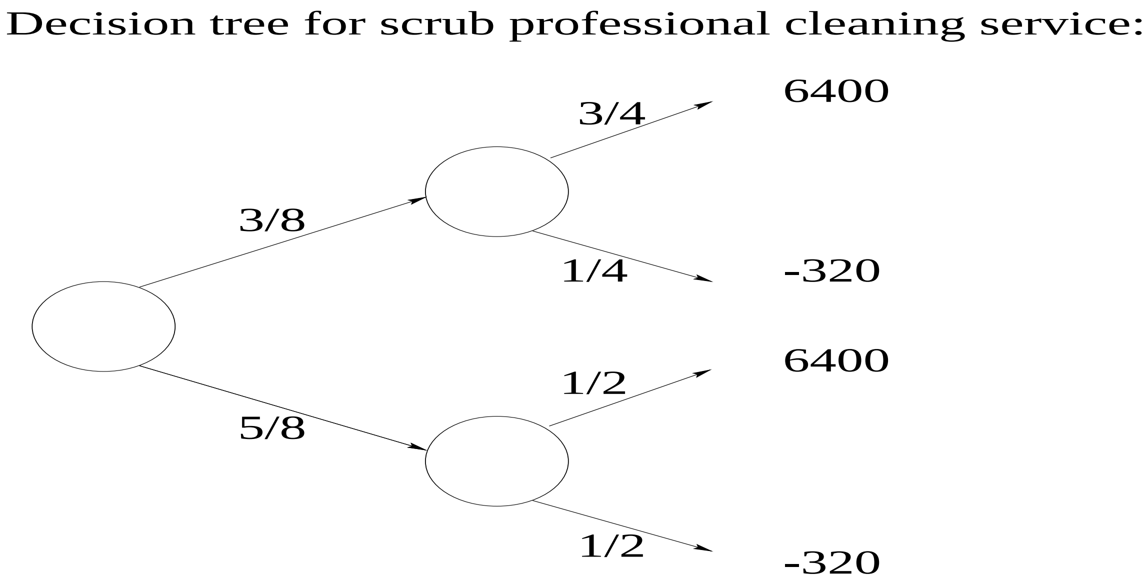
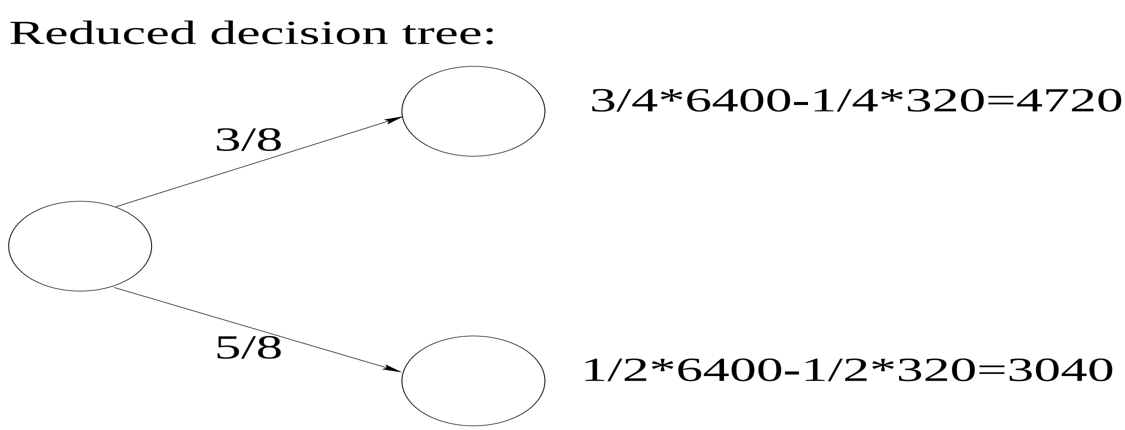
45-760: Quantitative Methods for the Management Sciences
Prof. G. Cornuejols
Prof. M. Trick

Solutions for Exercises 73-85

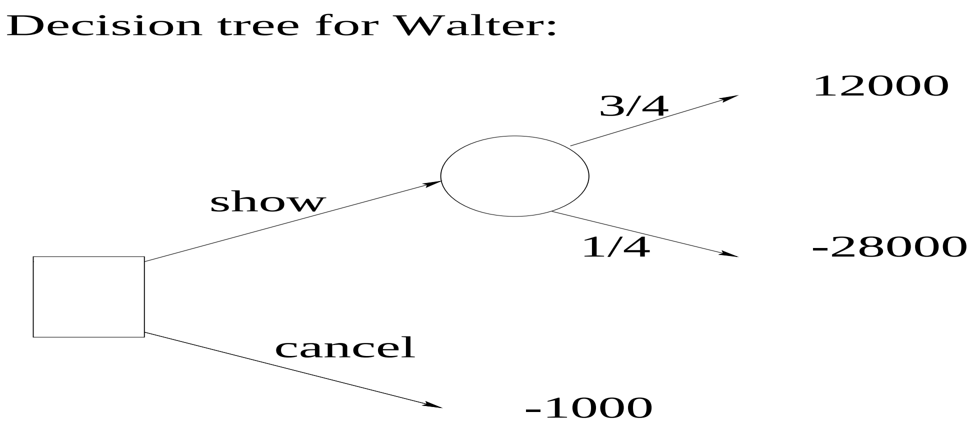
Solution to Exercise 73 *The decision tree for scrub professional cleaning service:*

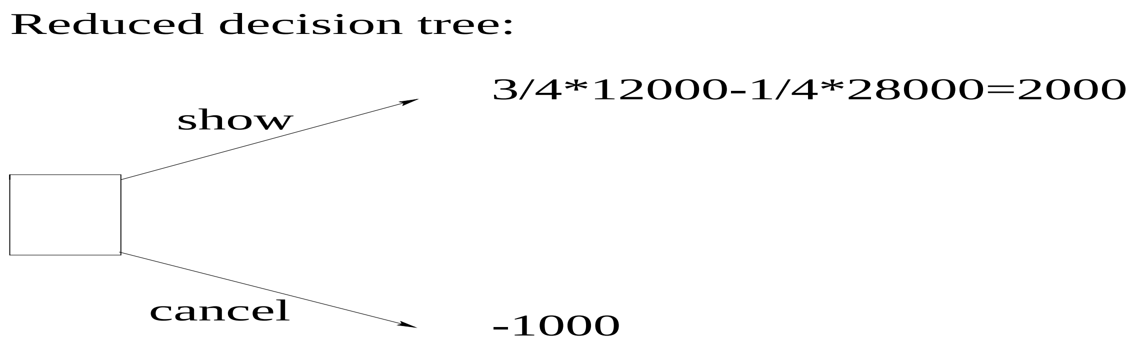




Therefore, the expected return is$= 3/8 * 4720 + 5/8 * 3440 = 3920$

Solution to Exercise 74 *a) According to the decision tree given below Walter should show.*





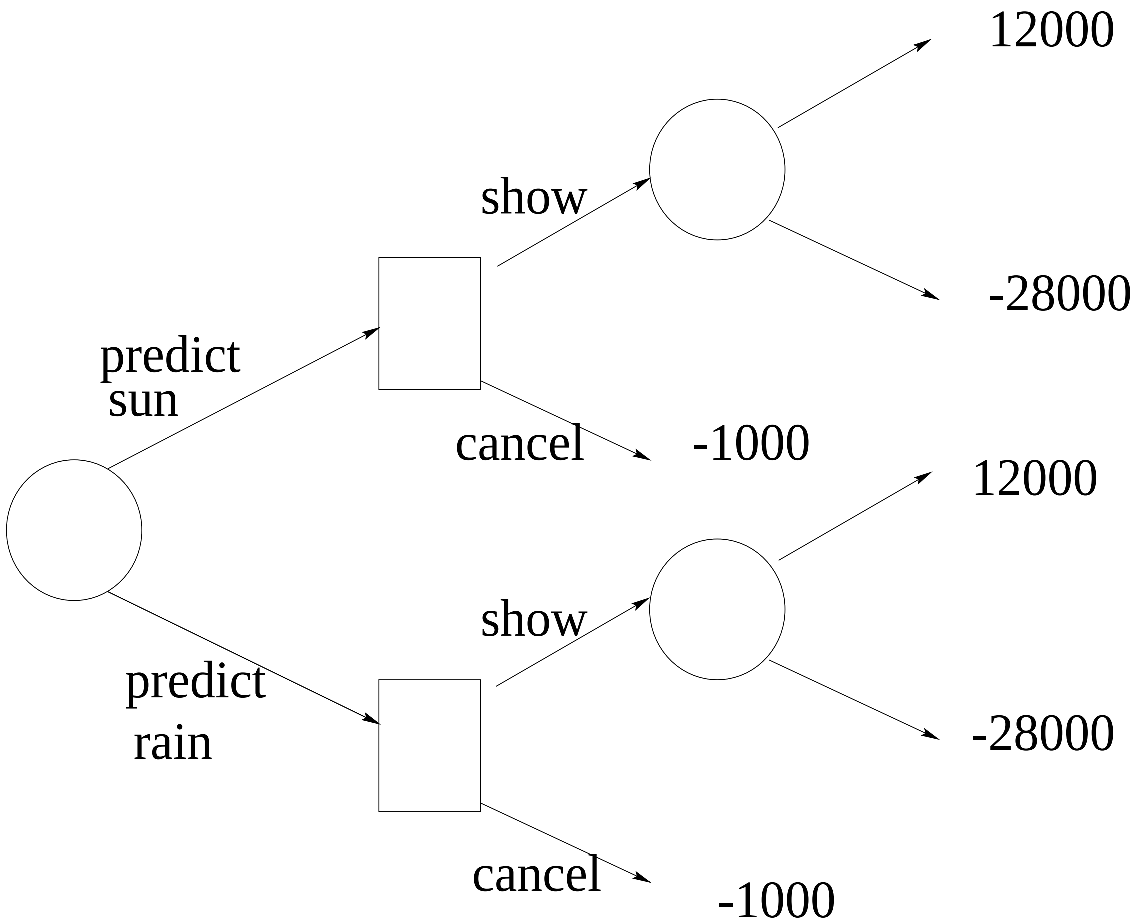
b) without information: expected net dollar return = 2000

with perfect information: expected net dollar return = 12000

EVPI=12000-2000=10000

c) If Walter had the forecast.

$$Prob(predicted\ sun) = Prob(predicted\ sun|sun)Prob(sun) + Prob(predicted\ sun|rain)Prob(rain)$$



$$
\begin{aligned}
Prob(sun|predicted\ sun) &= \frac{Prob(predicted\ sun|sun)Prob(sun)}{Prob(predicted\ sun)} \\
&= \frac{0.8 * 3/4}{(0.8 * 3/4 + 0.1 * 1/4)} \\
&= 0.96
\end{aligned}
$$

$$
\begin{aligned}
Prob(rain|predicted\ sun) &= 1 - 0.96 = 0.04 \\
Prob(rain|predicted\ rain) &= \frac{0.9 * 1/4}{(0.9 * 1/4 + 0.2 * 3/4)} = 0.6 \\
Prob(sun|predicted\ rain) &= 1 - 0.6 = 0.4
\end{aligned}
$$

If predicted sun:

show: expected value$=0.96*12000-0.04*28000=10400$

cancel: expected value $=-1000$

If predicted rain:

show: expected value$=0.4*12000-0.6*28000=-12000$

cancel: expected value $=-1000$

The strategy he should follow is: If predicted sun, then show. If predicted rain, then cancel.

d)

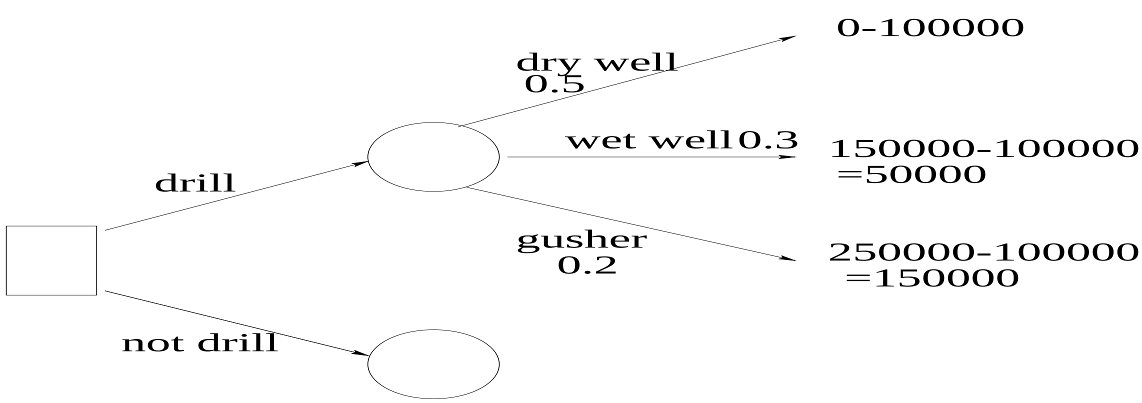
$$Prob(predicted\ sun)=0.8*3/4+0.1*1/4=2.5/4$$

$$Prob(predicted\ rain)=1-2.5/4=1.5/4$$

$$EVSI=2.5/4*10400+1.5/4*(-1000)-2000=4125$$

Walter is willing to pay less than \$ 4125.

Solution to Exercise 75 *a) The decision tree for wildcat oil:*



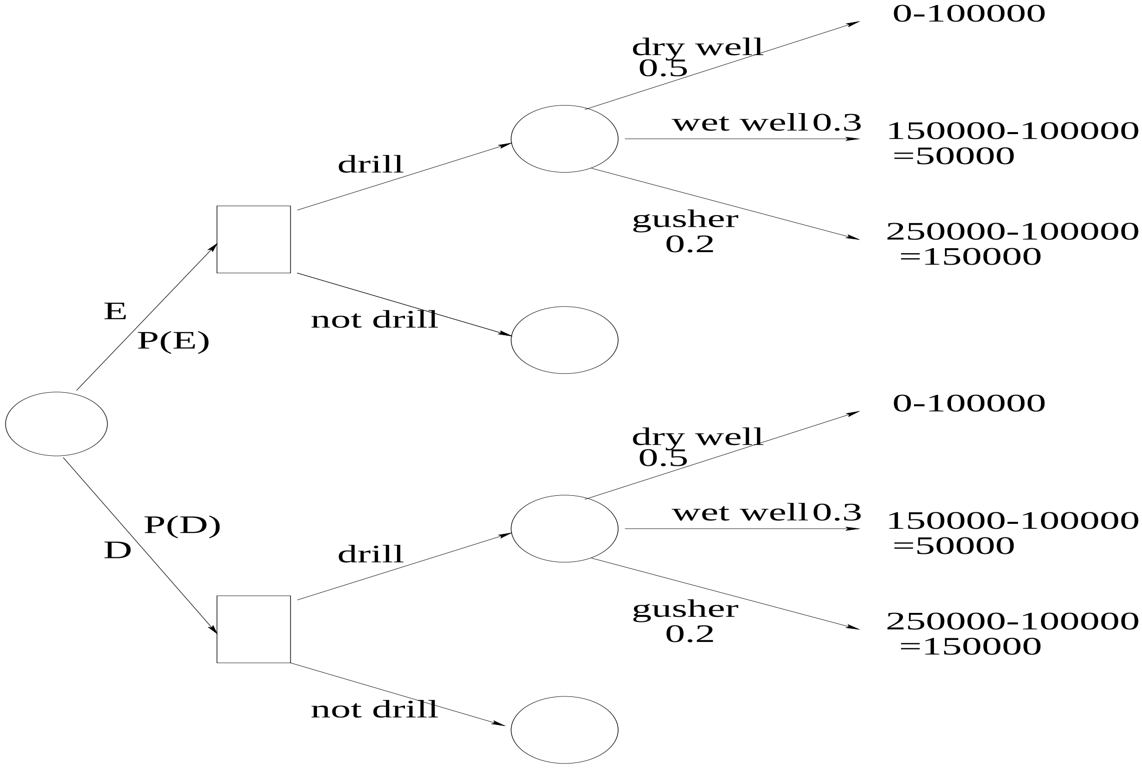
b) if drill: expected profit$=0.5*(-100000)+0.3*50000+0.2*150000=-5000$

Therefore, should not drill.

c) $EVPI=0.2*150000+0.3*50000-0=45000<50000$

Therefore, Wildcat should not accept their offer.

d) Decision tree for Wildcat oil:



$$Prob(E) = Prob(E|Dry)P(Dry)+P(E|W)P(W)+P(E|G)P(G) = 0.5/3+3/4*0.3+1*0.2 = 0.592$$

$$Prob(Dry|E) = \frac{Prob(E|Dry)Prob(Dry)}{Prob(E)} = \frac{0.5/3}{0.592} = 0.28$$

$$Prob(W|E) = \frac{0.9/4}{0.592} = 0.38$$

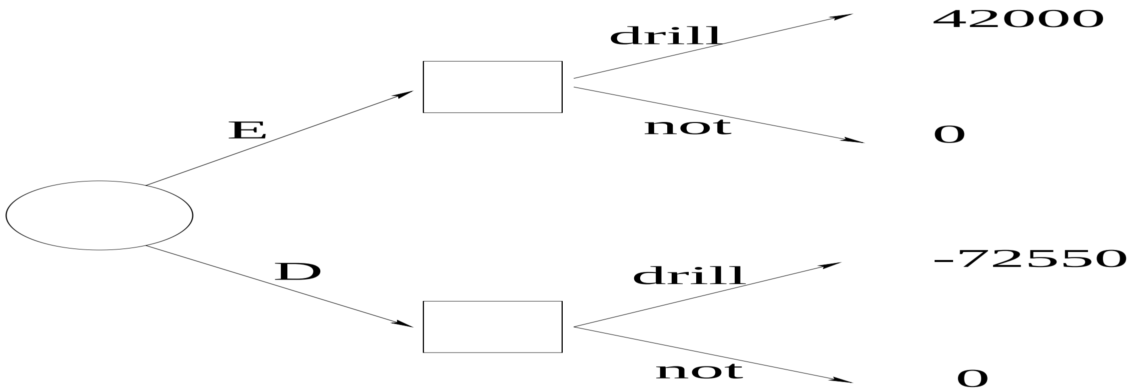
$$Prob(G|E) = 1 - 0.28 - 0.38 = 0.34$$

$$Prob(Dry|D) = \frac{Prob(D|Dry)Prob(Dry)}{Prob(D)} = \frac{0.5 * 2/3}{(1 - 0.592)} = 0.817$$

$$Prob(G|D) = 0$$

$$Prob(W|D) = 1 - 0 - 0.187 = 0.183$$

The reduced decision tree will be:



$$EVSI = P(E) * 42000 + P(D) * 0 - 0 = 24864$$

Wildcat should pay less than $ 24864

Solution to Exercise 76 *a)*

P_2

		G	E
P_1	G	(3,3)	(6,10)
	E	(10,6)	(5,5)

There are two pure strategy equilibria: (G,E) and (E,G) and one mixed strategy equilibrium, where fisherman 1's pobabilities are $(p_1, p_2) = (1/8, 7/8)$ *and fisherman 2's probabilities are* $(q_1, q_2) = (1/8, 7/8)$ *and the payoff is* $(5.625, 5.625)$

b) By having such a geographical constraint imposed, the options of the fisherman become more limited. In this case, such a restriction would likely constrain each fisherman to fish in different places (assuming they have different geographical origins), hence forcing either (G,E) or (E,G) (but not both).

Solution to Exercise 77 *Let* (x_1, x_2, x_3) *be P1's probabilities and let* (y_1, y_2, y_3, y_4) *be P2's probabilities. Then the optimal startegy is:*

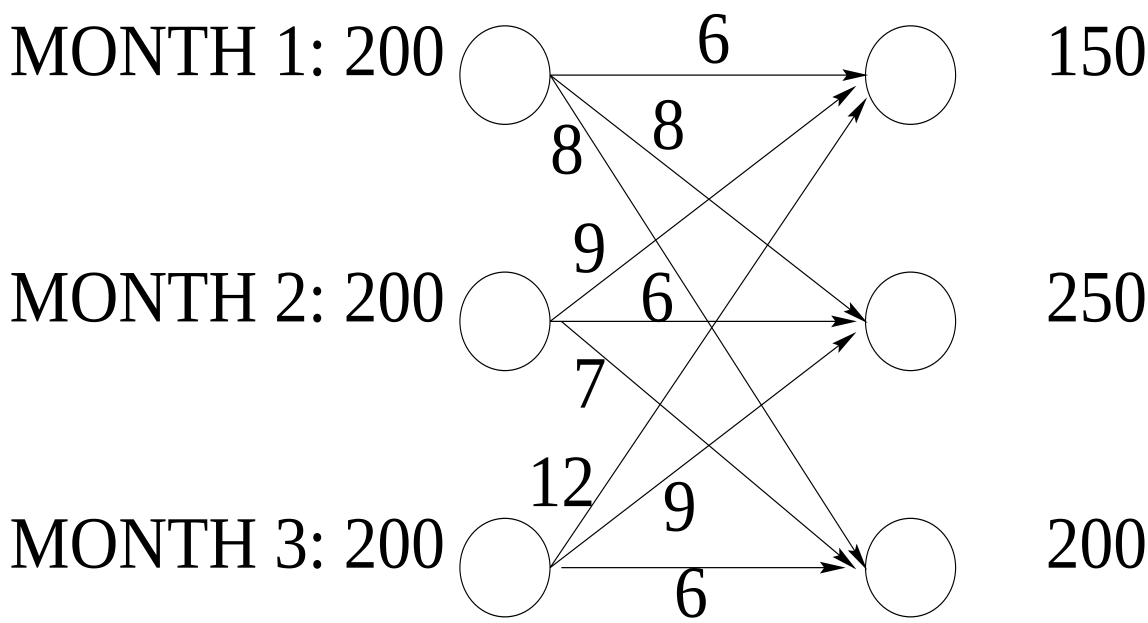
$(x_1, x_2, x_3) = (0.57, 0.43, 0)$,

$(y_1, y_2, y_3, y_4) = (0.71, 0, 0.29, 0)$

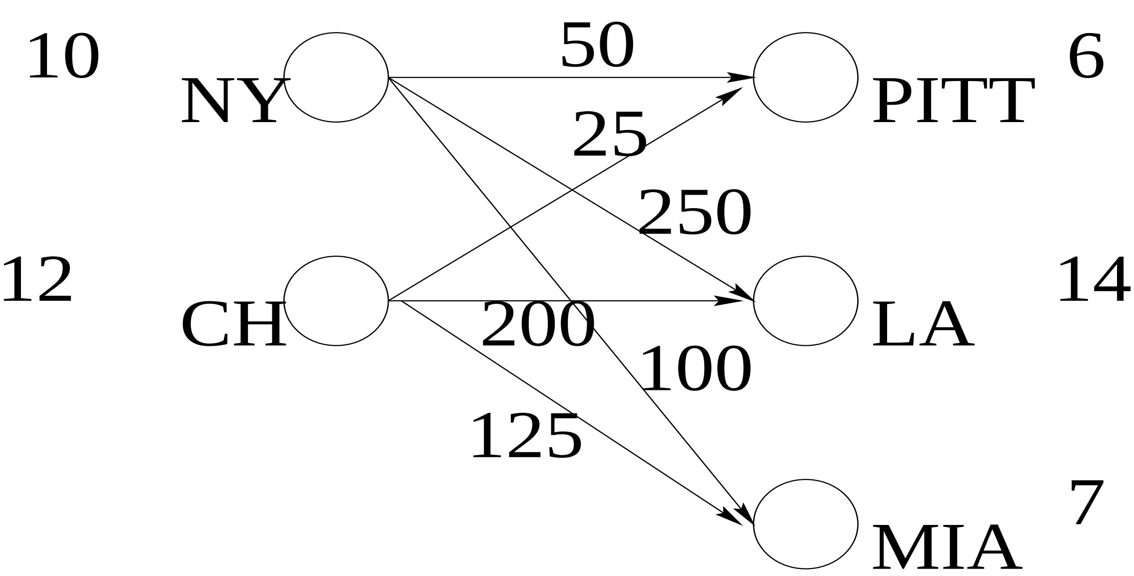
with payoff $(3.14, -3.14)$

Solution to Exercise 78

Solution to Exercise 79 *Find a minimum cost feasible flow.*



Solution to Exercise 80 *The diagram is given below:*



Solution to Exercise 81 *a)*

$MAX \sum_{i=A}^{D} \sum_{j=E}^{H} c_{ij} x_{ij}$

subject to

$$\begin{array}{rcll} \sum_{j=E}^{H} x_{ij} & = & 1 & , \quad i = A, B, C, D \\ \sum_{j=A}^{O} x_{ij} & = & 1 & , \quad i = E, F, G, H \\ x_{ij} & \geq & 0 & , \quad \forall i, j \end{array}$$

b) This is an assignment problem, which is a special case of the transportation poblem. Since the supplies and demands are integral (they are equal to 1), every basic solution will be integral. This plus the fact that all rows and columns sum up to one imply the marriage theorem.

c) No- the polyhedron described by the constraints will no longer have exclusively integral extreme points, hence the optimal solution may not be integral. (Note that this implies that the problem will have lost its network structure.)

Solution to Exercise 82 *a) Here the supply nodes are cash, bond 1, bond 2 and bond 3 supplies. The demand nodes are the cash demands for each month. The arc costs correspond to the penalties of early sale. To make the problem balanced, a dummy demand node with a demand of \$ 120 must be added (with all incoming arcs having zero cost).*

b) See diagram.

Solution to Exercise 83 *See diagram.*

Solution to Exercise 84 *See diagram.*

Solution to Exercise 85 *a) This is a DEA analysis for player 3 (Johnson). His efficiency rating is $\theta = 0.703$, which is strictly less than one, so he is inefficient. The virtual producer is*

$$VP = 0.55(Martin : P1) + 0.697(Polcovivh : P2)$$

so that VP's input is:

$$0.55(135) + 0.697(82) = 131.4 \text{ at bats}$$

and its outputs are:

$$0.55(41) + 0.697(25) = 40.0 \textit{ hits}$$

$$0.55(6) + 0.697(1) = 4.0 \textit{ home runs}$$

b) The alternate LP is

$MAX 40u_1 + 4u_2$

subject to

$$\begin{array}{lccccccc}
187v & & & & & = & 1 \\
-135v & + & 41u_1 & + & 6u_2 & \leq & 0 \\
-82v & + & 25u_1 & + & u_2 & \leq & 0 \\
-187v & + & 40u_1 & + & 4u_2 & \leq & 0 \\
v & , & u_1 & , & u_2 & \geq & 0
\end{array}$$